and Breach, New York, 1968, pp. 165–266, Chap. 3.

[2]Fiveland, W. A., "A Discrete-Ordinates Method for Predicting Radiative Heat Transfer in Axisymmetric Enclosure," American Society of Mechanical Engineers, Paper 82-HT-20, 1982.

[3]Raithby, G. D., and Chui, E. H., "A Finite-Volume Method for Predicting a Radiative Heat Transfer in Enclosures with Participating Media," *Journal of Heat Transfer*, Vol. 112, May 1990, pp. 415–423.

[4]Chui, E. H., Raithby, G. D., and Hughes, P. M. J., "Prediction of Radiative Transfer in Cylindrical Enclosures with the Finite-Volume Method," *Journal of Thermophysics and Heat Transfer*, Vol. 6, No. 4, 1992, pp. 605–611.

[5]Chui, E. H., Hughes, P. M. J., and Raithby, G. D., "Implementation of the Finite-Volume Method for Calculating Radiative Transfer in a Pulverized Fuel Flame," *Combustion Science and Technology*, Vol. 92, 1993, pp. 225–242.

[6]Baek, S. W., and Kim, M. Y., "Analysis of Radiative Heating of a Rocket Plume Base with the Finite-Volume Method," *International Journal of Heat and Mass Transfer*, Vol. 40, No. 7, 1997, pp. 1501–1508.

[7]Baek, S. W., and Kim, M. Y., "Modification of the Discrete-Ordinates Method in an Axisymmetric Cylindrical Geometry," *Numerical Heat Transfer, Part B*, Vol. 31, No. 3, 1997, pp. 313–326.

[8]Chai, J. C., Parthasarathy, G., Lee, H. S., and Patankar, S. V., "Finite Volume Radiative Heat Transfer Procedure for Irregular Geometries," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 3, 1995, pp. 410–415.

[9]Moder, J. P., Chai, J. C., Parthasarathy, G., Lee, H. S., and Patankar, S. V., "Nonaxisymmetric Radiative Transfer in Cylindrical Enclosures," *Numerical Heat Transfer, Part B*, Vol. 30, 1996, pp. 437–452.

[10]Kaminski, D. A., "Radiative Transfer from a Gray, Absorbing-Emitting, Isothermal Medium in a Conical Enclosure," *Journal of Solar Energy Engineering*, Vol. 111, Nov. 1989, pp. 324–329.

[11]Kim, M. Y., and Baek, S. W., "Prediction of Radiative Heat Transfer in a Three-Dimensional Gas Turbine Combustor with the Finite-Volume Method," *Transactions of the Korean Society of Mechanical Engineers (B)*, Vol. 20, No. 8, 1996, pp. 2681–2692 (in Korean).

Acoustically Levitated Droplet Viscosity Measurement

G. F. Mitchell,* Y. Bayazitoglu,† and R. Shampine‡
Rice University, Houston, Texas 77251-1892

I. Introduction

THE ability to levitate a sample and isolate it from contact with a solid surface prevents the contamination of the sample, which could alter the measured value of the property of interest, and also eliminates surface imperfections that can cause premature solidification in a subcooled liquid sample.[1] While the measurement of surface tension and viscosity are often mentioned together, the theory concerning the containerless measurement of these properties has been seen to diverge somewhat.[2] Chandrasekhar[3] and Reid[4] extended the analysis to permit arbitrary droplet viscosity. These works, however, considered only cases where the viscosity of the host medium surrounding the droplet could be neglected. Miller and Scriven[5] then considered a viscous spherical droplet oscillating in a medium of arbitrary viscosity. More recently, the normal-mode analysis of Prosperetti[6] and the numerical studies of Cumins and Blackburn[7] and Bayazitoglu and Suryanarayana[8–10] have extended the range of theory closer to areas of experimental interest as technological advancements have allowed the measurements of quantities that were once beyond reach.

Earth-based experiments necessarily combine droplet viscosity and static deformation. Nonetheless, containerless surface tension measurements of low-viscosity liquids using Earth-based levitators have obtained good results,[11,12] both in terms of measuring the surface tension and in confirming theoretical predictions for the frequency splitting of deformed droplets.

The viscosity of the droplet serves as a damping force for the oscillations that decay exponentially as $e^{-\beta t}$. Here, β is a complex-valued decay factor, where the real part τ^{-1} is the damping rate and the imaginary part ω corresponds to the natural frequency of oscillations. Theoretical relations between τ, ω, and the viscosity ν exist,[8,13] which allow the viscosity of a liquid to be deduced from the damping rate of droplet oscillations just as natural frequency measurements have been used to find surface tension.

To this point, levitation experiments have focused on properties besides viscosity, which is a much more technically challenging property to measure using levitation. While damping rates have been measured for oscillating viscous droplets suspended in an immiscible liquid of comparable viscosity,[14] such data for liquid droplets oscillating in air have not appeared in the literature.[1] Trinh et al.[14] found discrepancies between the measured and theoretical damping rates that were attributed to the inertia of the viscous boundary layer and temporal variation in the fluid properties near the interface. The present work seeks to obtain damping rate data for liquid-air systems, thereby simplifying the analysis and reducing the inertia of the viscous boundary layer by reducing its thickness. Typical assumptions made for theoretical prediction are isothermal droplets making small-amplitude oscillations about a spherical shape in the absence of external forces and contamination. The present experimental work is intended primarily to test the feasibility of the concepts involved. It should be noted that the droplets are axisymmetric, but not spherical. The levitation forces cause them to be slightly flattened axisymmetrically.

Damping rate information may be obtained through measurement of the decay time or by measuring the sharpness of a resonant peak in the frequency spectrum. Using these methods, three different procedural approaches are possible. The time required for decay could be taken from an oscilloscope trace of the decaying oscillation amplitude immediately following the cessation of the driving force. This approach was used by Trinh et al.[14] in their study of droplets immersed in an immiscible liquid. In their case, however, the natural frequencies were much lower and the liquid host medium provided additional damping.

The damping rate could also be derived from the sharpness or quality factor, Q, of a resonant peak in the response spectrum of the droplet. This could be obtained simply by slowly sweeping the excitation frequency through a natural frequency while measuring the amplitude of the droplet response. This method is also cited by Trinh et al. in their work in immiscible liquid systems. They show the swept frequency response of a silicone/CCl_4 droplet levitated in distilled water, and the resemblance to a bell curve is easily seen. However, it should be noted that a droplet immersed in an immiscible liquid takes on a virtually spherical shape even in gravity caused by the small difference in density between the two liquids. As such, each mode of oscillation exhibits just one resonant peak, in

Received Dec. 23, 1997; revision received May 14, 1998; accepted for publication May 14, 1998.

*Graduate Student; currently Research Engineer, Production Operations Division, Exxon Production Research Company, Houston, TX 77252-2189.

†Professor, Department of Mechanical Engineering and Materials Science, George R. Brown School of Engineering. Member AIAA.

‡Graduate Student; currently Development Engineer, Wellsite Intervention Services, Dowell, Schlumberger, Rosharon, TX 77583-1590.

contrast to the three peaks expected from a deformed droplet. In addition, the required rate of frequency sweeping is so low that the natural frequency shift caused by droplet evaporation[15] becomes a concern. Trinh et al. report a sweep rate of 10 mHz/s as the maximum for which the swept frequency response matches the response at various discrete frequencies. In addition, it has been noted that droplets are very sensitive to fluctuations in the sweep rate; sudden changes induce artificially impulsive increases in amplitude.

The means for obtaining the frequency response spectrum in this experiment uses an electrical engineering analogy in viewing an oscillating droplet as a bandpass filter. The filter input is the excitation signal, whereas the output is the oscillatory response of the droplet. Because droplets are steadily evaporating (allowing measurements for multiple sizes from a single levitated sample), it is desirable to measure the entire response spectrum at once. To do this, the input signal must contain equal energy at every frequency within the range of interest (~50–150 Hz). This signal is reflected to as white noise. Using white noise, the Fourier transform of the output signal is the frequency response of the droplet.

II. Apparatus and Procedure

The apparatus used for this experiment, which is shown in Fig. 1, is essentially identical to that used in a previously published surface tension experiment.[11] A 20-kHz piezoelectric transducer is driven at its resonant frequency, causing a high-intensity pressure wave to radiate from the tip of an attached aluminum horn. A small concave reflector situated one and one-quarter wavelengths above the horn causes a standing wave with two distinct nodes of locally minimized pressure to form. A small liquid droplet can be levitated slightly below such a node as a result of the net upward pressure force that balances the weight of the droplet. If the 20-kHz signal that provides the levitating force is modulated, variations in the intensity of the levitating force are realized by the droplet.

The oscillatory response of the droplet is monitored using a simple and compact photodetector that registers the change in the intensity of light passing from a small infrared light emitting diode (LED) to a detector situated opposite the droplet. For small oscillations, the amplitude of the photodetector output is taken to be proportional to the amplitude of droplet oscillations. The modulation frequency at which the amplitude of the photodetector output is maximized is recorded as a natural frequency. Instead of being viewed directly on an oscilloscope, the output from the photodetector is sent to a computerized data acquisition (DAQ) and analysis package that samples the signal and performs a spectral analysis to yield the response spectrum of the droplet. In addition, a photograph of the cross section is taken from the side for size measurement purposes. Assuming a droplet of ellipsoidal cross section, the radius of a sphere having the same volume as the droplet is

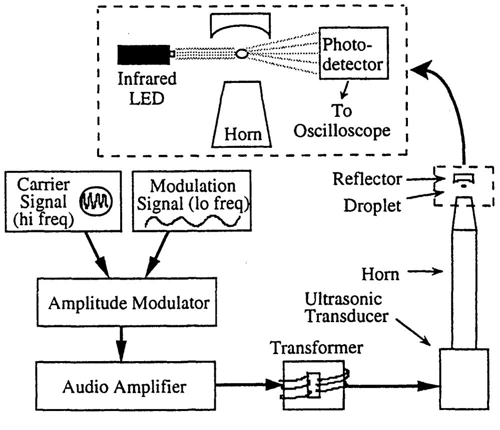

Fig. 1 Acoustic levitator.

easily found from the semimajor and semiminor axes a and b of the sideways projection of the droplet using

$$R = (a^2b)^{1/3} \tag{1}$$

The spectrum is filtered to eliminate noise and then stored for analysis. Likewise, the procedure utilized in this experiment is essentially identical to that described in Bayazitoglu and Mitchell.[11] In this experiment, though, quantitative spectral data are being collected. Care must be taken in setting up the DAQ system to achieve a balance between the upper frequency limit of the spectrum, the frequency resolution, and the time required to collect and analyze the samples. This experiment uses a sampling duration of 5 s, which yields a frequency resolution of 0.2 Hz.

The droplet response spectrum is quite noisy, but can be significantly reduced by using low-pass filtering before the DAQ, and frequency domain smoothing (median filter) to ease peak width measurements. The spectrum shows a spike at 60 Hz from ac line voltage. (The experiment must be performed in a darkened room to keep this from swamping the signal of interest.) The next higher frequency peak is a translational (mode 1) resonance.[11] The next three peaks are the three split frequencies. The two highest frequencies are seen to be much closer together than any of the others, and this closeness often makes them difficult to distinguish. Indeed, for many droplets a third distinct peak cannot be seen at all because of its proximity to another peak. The lowest of these three frequencies is used for determining viscosity.

III. Analysis

The droplets considered in this work undergo small-amplitude deviations from the equilibrium shape. As described elsewhere,[12] the acoustic levitator only excites oscillations of the fundamental ($l = 2$) mode. Translational (mode 1) frequencies, while often present because of instability, are ignored. Hence, a droplet in this experiment may be considered as a linear oscillator with one degree of freedom (momentarily neglecting the multiplicity of natural frequencies that arise from the static deformation). The response of this system may be measured using time-domain (decay rate) or frequency-domain techniques.

The frequency response of the droplet exhibits one to three discernible peaks, depending on the static deformation of the droplet. For any peak, the value of τ^{-1} is easily found from the width of the peak; the difference between the frequencies on either side of the peak at which the amplitude of oscillation is 3 dB ($1/\sqrt{2}$) below the maximum amplitude. From Thomson[16]

$$\tau^{-1} = (\omega_2 - \omega_1)/2 \tag{2}$$

where ω_1 and ω_2 are the frequencies above and below the peak, respectively. Equation (2) enables the determination of the free oscillation decay rate from the response of the droplet to the white noise forcing function.

The analysis of Suryanarayana and Bayazitoglu[8] allows us to find the following expression for the damping rate:

$$\tau^{-1} \approx 4.865\nu/R^2 = 0.04875/R^2 \tag{3}$$

This is plotted in Fig. 2 for the range of interest.

IV. Results and Discussion

The data collected in the viscosity experiment for distilled water are summarized in Table 1. The raw data were reduced using Eq. (2) to obtain the experimental values of τ^{-1}, which are plotted against droplet radius in Fig. 2 along with analytical predictions from Suryanarayana and Bayazitoglu. The predictions of Marston's paper[13] are also shown.

Table 1 Viscosity experiment data for distilled water at 24°C

R_{sphere}, cm	ω	Peak width, Hz	τ^{-1}, rad/s	ν_{exp}
0.0796	198.8	1.573	4.9418	0.00644
0.1140	110.0	1.072	3.3661	0.00900
0.1121	114.7	1.345	4.2256	0.01091
0.0894	176.9	1.980	6.2204	0.01022
0.0986	117.6	1.294	4.0664	0.00812
0.1241	83.8	0.821	2.5783	0.00816
0.1209	92.0	1.084	3.4059	0.01023
0.1137	94.8	1.044	3.2786	0.00870
0.0844	146.2	1.480	4.6496	0.00680
0.0806	159.7	2.090	6.5659	0.00877
0.1232	97.9	0.937	2.9444	0.00918
0.1160	107.1	1.040	3.2673	0.00904
0.1127	118.2	1.690	5.3093	0.01386
0.1013	117.2	1.440	4.5239	0.00954
0.0993	142.8	1.700	5.3407	0.01082

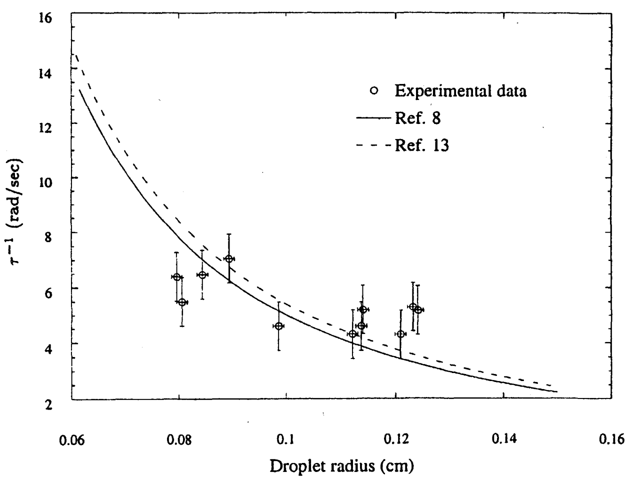

Fig. 2 Plot of damping rate τ^{-1} vs droplet radius for distilled water.

As shown in the uncertainty analysis, the most significant source of uncertainty in the containerless measurement of viscosity is the measurement of the peak width $\omega_2 - \omega_1$ in the frequency spectrum because the error incurred here is propagated throughout the rest of the calculations. The peak width measurement would benefit from greater frequency resolution, but this would require a longer sampling duration, which is undesirable because of the prospect of blurring the spectrum as a result of the frequency shift from evaporation.

The mean value of viscosity for distilled water at 24°C found in this experiment was 0.00932 ± 0.00181 cS. This agrees well with the published value of 0.009202 cS.[17] The experimentally measured viscosity lies above the expected value, but to a lower degree than is noted by Trinh et al.,[14] who suggests that the discrepancy is because of the additional dissipation resulting from fluid circulation induced within the droplet because of the levitating force and the viscous boundary layer. In their work, of course, the host medium is a liquid, thereby making the viscous boundary layer much thicker than for a liquid leviating in air.

V. Conclusions

Experimental observations and numerical data concerning the damping rates of acoustically levitated oscillating droplets are presented. The focus in this study has been on ellipsoidal liquid droplets of relatively low, albeit finite, viscosity.

This experiment collected damping rate data in an effort to obtain containerless measurements of viscosity. Although considerable scatter is present in the limited results presented here, the expected result lies within the bounds of the experimental uncertainty and good qualitative agreement is obtained with two different theoretical predictions.[8,13] Also, a new method of exciting droplet oscillations for frequency spectrum measurements was successfully tested.

Comparisons between experimental data and analytical predictions of the damping rates of deformed levitating droplets are instructive for evaluating the state of knowledge of the forces acting on an acoustically levitated droplet. If the influence of the modulated acoustic standing wave on the droplet is known, then its impact on observed thermophysical properties may be evaluated as well. For now, however, fairly accurate results have been realized from a simple, inexpensive, and compact apparatus.

Many possibilities exist for applying the work described here. In particular, levitation techniques have great potential in facilitating study of the dependence of viscosity on quantities such as temperature, pressure, and even the concentration of added contaminants, whether dissolved or suspended in solution. In addition, undercooled liquids may be studied much more easily when levitated because the lack of a solid contact surface eliminates preferred nucleation sites. Hence, greater subcooling may be achieved for longer periods of time. However, perhaps the most significant application of the work in acoustic levitation will be to pave the way for advancements in electromagnetic levitation, where molten metals provide significantly greater technical challenges in thermophysical property measurement.

References

[1]Egry, I., and Sauerland, S., "Containerless Processing of Undercooled Melts: Measurements of Surface Tension and Viscosity," *Materials Science and Engineering*, Vol. A178, No. 1–2, 1994, pp. 73–76.

[2]Barmatz, M., "Overview of Containerless Processing Technologies," *Material Processing in the Reduced Gravity Environment of Space*, edited by G. E. Rindone, North–Holland, New York, 1982, pp. 25–36.

[3]Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability*, Clarendon, Oxford, England, UK, 1961, pp. 467–480.

[4]Reid, W. H., "Oscillations of a Viscous Liquid Droplet," *Quarterly of Applied Mathematics*, Vol. 18, No. 1, 1960, pp. 86–89.

[5]Miller, C. A., and Scriven, L. E., "The Oscillations of a Fluid Droplet Immersed in Another Liquid," *Journal of Fluid Mechanics*, Vol. 32, No. 3, 1968, pp. 417–435.

[6]Prosperetti, A., "Normal-Mode Analysis for the Oscillations of a Viscous Liquid Drop in an Immiscible Liquid," *Journal de Mecanique*, Vol. 19, No. 1, 1980, pp. 149–182.

[7]Cummings, D. L., and Blackburn, D. A., "Oscillations of Magnetically Levitated Aspherical Droplets," *Journal of Fluid Mechanics*, Vol. 224, March 1991, pp. 395–413.

[8]Suryanarayana, P. V. R., and Bayazitoglu, Y., "Surface Tension and Viscosity from Damped Free Oscillations of Viscous Droplets," *International Journal of Thermophysics*, Vol. 12, No. 1, 1991, pp. 137–151.

[9]Suryanarayana, P. V. R., and Bayazitoglu, Y., "Effect of Static Deformation and External Forces on the Oscillations of Levitated Droplets," *Physics of Fluids A*, Vol. 3, No. 5, 1991, pp. 967–977.

[10]Bayazitoglu, Y., and Suryanarayana, P. V. R., "Dynamics of Oscillating Viscous Droplets Immersed in Viscous Media," *Acta Mechanica*, Vol. 95, No. 1–4, 1992, pp. 167–183.

[11]Bayazitoglu, Y., and Mitchell, G., "Experiments in Acoustic Levitation: Surface Tension Measurement of Deformed Droplets," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 4, 1995, pp. 694–702.

[12]Trinh, E., Marston, P. L., and Robey, J. L., "Acoustic Measurement of the Surface Tension of Levitated Drops," *Journal of Colloid and Interface Science*, Vol. 124, No. 1, 1988, pp. 95–103.

[13]Marston, P. L., "Shape Oscillation and Static Deformation of Drops and Bubbles Driven by Modulated Radiation Stresses—Theory," *Journal of the Acoustical Society of America*, Vol. 67, No. 1, 1980, pp. 15–26.

[14]Trinh, E., Zwern, A., and Wang, T. G., "An Experimental Study of Small-Amplitude Drop Oscillations in Immiscible Liquid Systems," *Journal of Fluid Mechanics*, Vol. 115, Feb. 1982, pp. 453–474.

[15]Daidzic, N., Stadler, R., and Dominick, J., "Experimental Techniques for Measurement of Droplet Evaporation," *Proceedings of the 8th International Conference on Liquid Atomization and Spray Systems*, National Inst. of Standards and Technology, Rouen, France, 1994, pp. 875–882 (ICLASS-94).

[16]Thomson, W. T., *Theory of Vibration with Applications*, 2nd ed., Prentice–Hall, Englewood Cliffs, NJ, 1981.

[17]Lide, D. R., *Handbook of Chemistry and Physics*, 70th ed., CRC Press, Boca Raton, FL, 1990.

Small Cavitating Venturi Performance Characteristics at Low Inlet Subcooling

I. Y. Chen,* S. G. Liou,* and J. S. Sheu†
National Yunlin University of Science and Technology, Yunlin 640, Taiwan, Republic of China

Introduction

CAVITATING venturis (CVs) are used to provide a passive flow control in many liquid flow systems. When liquid flow cavitates at the throat of a venturi, it can provide a choked flow regime useful for flow control. If the liquid's inlet pressure and temperature are fixed, a wide range of downstream pressures may be imposed with no effect on flow.[1] Under normal operation, the cavitating venturi is designed to provide cavitation at the throat so that the flow rate is independent of the downstream pressure. With cavitation, the flow at the throat is choked and the throat pressure (P_{th}) is equal to the saturation pressure at the inlet temperature [$P_{sat}(T_{in})$]. In typical applications, the CV inlet subcooling, dP_{sub}, which is defined as the pressure difference between the inlet pressure (P_{in}) and $P_{sat}(T_{in})$, is typically in the order of 700–1400 kPa.

There was a special application for CVs in the two-phase ammonia active thermal control system on the U.S. Space Station Freedom (SSF) in 1990. Here, the CVs were intended to be operated under conditions that were quite different from the typical applications. These CVs had very low inlet subcooling (40–140 kPa) and very small throat diameters. Prototypic CVs were included in the ground test article (GTA), a prototypic SSF active thermal control system (ATCS) two-phase thermal loop. During operation of the GTA in 1991, an anomalous CV overflow behavior was observed. Here the flow is a liquid nonchoked flow throughout the venturi and flows at a higher mass flow rate than the cavitation limit of the choked flow rate. Following the discovery of CV overflow, a series of experiments were performed to gain understanding of the CV's overflow phenomenon and to define its limits.[2]

The overflow phenomenon of the CVs found in the GTA ammonia testing was understood after the testing. The generic phenomenon of CV overflow and recovery from overflow in various liquids was not well understood. Overflow would still be a problem for a system of parallel CVs with low inlet subcooling. The objective of this study was to conduct a test on small cavitating venturis with water at low inlet subcooling to observe the phenomenon of overflow. The results were correlated to provide understanding of the overflow phenomenon.

Received Nov. 24, 1997; revision received April 20, 1998; accepted for publication April 22, 1998.

*Associate Professor, Department of Mechanical Engineering.
†Graduate Student, Department of Mechanical Engineering.

Flow Equations and Relative Parameters

Normal Operation

A typical cavitating venturi is shown in Fig. 1. During normal operation, cavitation occurs at the throat, and the throat pressure is $P_{sat}(T_{in})$. Therefore, the CV's choked mass flow rate (M_c) can be determined from the Bernoulli's equation (assuming that the frictional loss in the convergent section is negligible) as

$$M_c = A_{th}[2\rho(P_{in} - P_{sat}(T_{in})]^{0.5} \tag{1}$$

where A_{th} is the CV throat area and ρ is the liquid density. For a nonchoked condition, the pressure drop is driven by the loss coefficient, K_v. The flow through the CV is all liquid, and K_v can be defined as $K_v = 2(P_{in} - P_{out})/(\rho V_{th}^2)$, where $V_{th} = M_{liq}/(\rho A_{th})$ is the throat liquid velocity, and P_{out} is the outlet pressure. K_v must be applied to account for the nonchoked flow rate, M_{nc}, through the venturi

$$M_{nc} = A_{th}[2\rho(P_{in} - P_{out})/K_v]^{0.5} \tag{2}$$

A dimensionless venturi pressure-difference ratio can be defined for both choked and nonchoked flows:

$$dP_r = [P_{out} - P_{sat}(T_{in})]/[P_{in} - P_{sat}(T_{in})] \tag{3}$$

The maximum dP_r for choked flow occurs where $Mc = M_{nc}$. This critical dP_r can be expressed as $dP_{r,crit} = 1 - K_v$. Under either the choked or nonchoked condition, the actual CV's measured mass flow rate (M_{act}) can be expressed in dimensionless form as the mass flow ratio, $M_r = M_{act}/M_c$. The mass flow ratio for a nonchoked liquid flow can be expressed as

$$M_r = M_{act}/M_c = [(1 - dP_r)/K_v]0.5 \tag{4}$$

Overflow Problem Description

In the classic theory of the operation of CV, cavitation is normally assumed to occur whenever dP_r is less than the critical value, $dP_{r,crit}$. Nonchoked flow is limited to $dP_r > dP_{r,crit}$ ($M_r < 1$). However, this is not always the case in the GTA tests. Once dP_r exceeds $dP_{r,crit}$, the flow in the CV becomes all

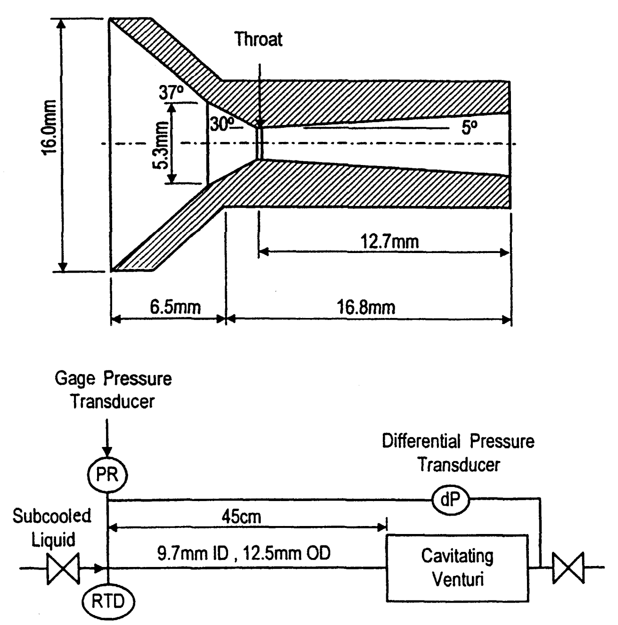

Fig. 1 Simplified schematic of CV's geometry and test section.